PHY121 Summer 2018
Problem Set #12
Due Friday 6/29

1. An Other Frame moves with speed $\beta = \frac{3}{5}$ in the $+x$ direction with respect to the Home Frame. In the Other Frame, an event is measured to occur at time $t' = 3$ s and position $x' = 1$ s. When and where does this event occur as measured in the Home Frame? Answer this question using a two-observer diagram and check your work with the appropriate Lorentz transformations.

2. Fred sits 65 ns west of the east end (therefore, 35 ns east of the west end) of a 100 ns long train station at rest on Earth. Sally operates a reference frame in a train racing east across the countryside at a speed $\beta = 0.5$. At a certain time ($t' = 0$) Sally passes Fred. At the same instant, Fred flashes a strobe lamp (event F), which sends bursts of light both east and west. Alan, standing at the west end of the station, receives the westward-traveling part of the flash (event A), and a bit later (according to the station clocks) Ellen, standing at the east end of the station, receives the eastward-traveling flash (event E).

   (a) When do events A and E occur in the station frame? Who sees the flash first (according to the station clocks), Alan or Ellen?

   (b) Draw a two-observer spacetime diagram of the situation, labeling the world-lines of Sally, Fred, Alan, Ellen, and the two light flashes. Plot and label events F, A, and E. Carefully draw and calibrate axes for Sally, in the rest frame of the train.

   (c) When and where do events A and E occur in Sally's frame? Sally claims that Ellen see the flash first in her frame. Is this true? Verify with the Lorentz transformation equations.

3. A very long measuring stick is placed in empty space at rest in an inertial frame (the stick frame). A spaceship of rest length L_R travels along the measuring stick at a speed $\beta = \frac{4}{5}$ relative to it. Two space cadets are each equipped with knives and synchronized watches and are stationed at rest on the ship frame at each end of the spaceship. At a prearranged time, the cadets simultaneously reach through their portholes and slice through the measuring stick.

   (a) How long is the spaceship according to the cadets?

   (b) How long is the spaceship according to observers along the measuring stick (i.e., at rest in the stick frame)?

   (c) Use a two-observer spacetime diagram to determine how long the cut portion of the stick is for an observer at rest in the stick frame. How much time passes between the two cuts in this frame?

(d) Use the Lorentz transformation equations to verify your result from part c.

(e) Explain in a short paragraph how two cadets only $\frac{3}{5}L_R$ apart can cut a hunk of measuring stick $\frac{5}{3}L_R$ long, if they cut simultaneously according to their synchronized watches.

4. A particle of mass m decays into two identical particles that move in opposite directions with speeds of $\frac{12}{13}$. What is the mass of each of the product particles (as a fraction of m)?

5. A spaceship with rest mass m_0 is traveling with an x velocity $v_{0,x} = \frac{4}{5}$ in the frame of the Earth. It collides with a photon torpedo (an intense burst of light) moving in the $-x$ direction relative to the Earth. Assume that the ship's shields totally absorb the photon torpedo.

   (a) The oncoming torpedo is measured by terrified observers on the ship to have an energy of $0.75m_0$. What is the energy of the photon torpedo in the frame of the Earth?

   (b) Determine the final x velocity (in the Earth's frame) and mass (in terms of m_0) of the damaged ship after it absorbs the torpedo. (Hint: Four-momentum is conserved.)