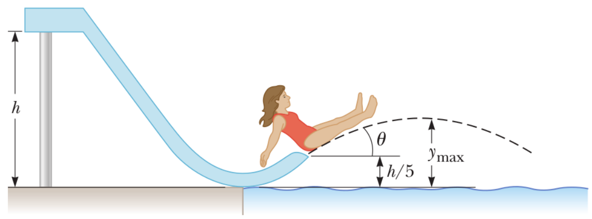
PHY121 Summer 2018
Problem Set #4
Due Thursday 5/31

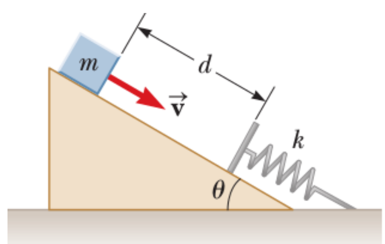
1. Consider a child on a slide. Assuming that she starts at the top of the slide, determine (in terms of the usual kinematic variables and h):

   (a) Her maximum height.

   (b) Her range before she plunges into the water.



2. An inclined plane of angle $\theta = 15°$ has a spring constant $k = 800$ N/m fastened securely at the bottom, so that the spring is parallel to the surface as shown. A block of mass $m = 5$ kg is place on the plane at a distance $d = 0.450$ m from the spring. From this position, the block is projected downward toward the spring with speed $v = 0.950$ m/s. How far is the spring compressed when the block momentarily comes to rest?



3. Find the center of mass of the following systems:

   (a) A square box of length L positioned such that one of its vertices is centered at our origin.

   (b) A square box of length L positioned such that one of its vertices is centered at our origin with a density $\rho(x, y, z) = 3(L - x)y^2$.

4. Find the final velocities of each of the objects in the following cases:

(a) A billiard ball moving at 4 m/s collides with another of equal mass at rest. The first ball moves with a final velocity of 3.2 m/s at an angle θ = 20°from its original direction of motion.

(b) A block of mass $3m$ is dropped vertically onto a car of mass $2m$ with horizontal velocity v = 6 m/s.

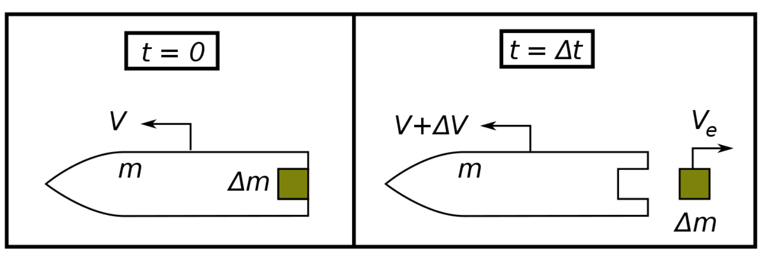
(c) A proton moving with a velocity of v_x = 150 m/s collides elastically with another proton that is initially at rest. The magnitude of the final velocity of each proton is equal.

(d) A car of mass m moving at speed v_1 = 30 m/s collides and couples with the back of a truck of mass $4m$ moving initially in the same direction with velocity v_2 = 20 m/s.

5. So far, we've studied momentum only as a change in velocity as a function of time. But momentum has two physical components, each of which can change as functions of time. For example, consider a rocket. Rockets are propelled by combining fuel in an explosive reaction which is propelled out the back of the rocket, changing the mass of the rocket in the process. Take a rocket of mass m, with momentum at time t of $p(t) = mv$. At a short time $t + dt$ later, the rocket has a mass $m + \Delta m$ and speed $v + dv$. The fuel ejected in time dt will have momentum $p_f = \Delta m(v - v_{ex})$, where the exhaust velocity v_{ex} is constant relative to the rocket. If the rocket loses mass, $m(t + dt) = m - dm$, $\Delta m = m(t + dt) - m(t) = -dm$. The total momentum of the system at time $t + dt$ is then

$$\begin{aligned} p(t+dt) = m_r v_r + m_f v_f = (m - (-dm))(v + dv) + (-dm)(v - v_{ex}) \\ = mv + vdm + mdv + dmdv - vdm + v_{ex}dm \\ = mv + mdv + v_{ex}dm \end{aligned}$$

where $dmdv$ is small enough to be negligible.



(a) In the absence of external forces, determine the velocity of the rocket after some time, as a function of initial mass m_0 and final mass m.

(b) A ship on *The Expanse* burns fuel at a rate of 1000 kg/s, with an exhaust velocity v_{ex} = 4000 km/s. How long would they need to burn to accelerate to 0.013 c, starting from rest? (This is the limit of classical mechanics. At a speed of 0.013 c, the error in the kinetic energy due to special relativity is $\approx$ 1%.)